

LESSON 6: Unit Rate Problems

[OBJECTIVE]

The student will solve unit rate problems involving unit pricing and constant speed using tape diagrams.

[PREREQUISITE SKILLS]

concept of ratio and ratio language, equivalent fractions, coordinate plane and plotting coordinates

[MATERIALS]

Student pages **S51 – S60**

Index cards – 5 per student pair

Toothpicks – 5 per student pair

Counters – 10 per student pair

Painter’s tape – 4 small pieces per student pair

[ESSENTIAL QUESTIONS]

1. How can I define a rate?
2. How can I use a tape diagram to show ratios?
2. How can I show a constant rate using a tape diagram?

[WORD WALL WORDS]

ratio, tape diagram, constant rate, unit pricing, unit rate, variable

[GROUPING]

Cooperative Pairs (CP), Whole Group (WG), Individual (I)

*For Cooperative Pairs (CP) activities, assign the roles of Partner A and Partner B to students. This allows each student to be responsible for designated tasks within the lesson.

[LEVELS OF TEACHER SUPPORT]

Modeling (M), Guided Practice (GP), Independent Practice (IP)

[MULTIPLE REPRESENTATIONS]

SOLVE, Graph, Verbal Description, Graphic Organizer, Pictorial Representation, Concrete Representation

[WARM-UP] (IP, I, WG) S51 (ANSWERS ARE ON T115.)

- Have students turn to S51 in their books to begin the Warm-Up. Students will be working with ratios and rates using tables. Monitor students to see if any of them need help during the Warm-Up. Have students complete the problems and then review the answers as a class. {**Verbal Description, Graphic Organizer**}

[HOMEWORK] (5 minutes)

Take time to go over the homework from the previous night.

[LESSON] [2 days – (1 day = 80 minutes) – M, GP, IP, WP, CP]

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SOLVE Problem**(WG, GP) S52 (Answers on T116.)**

Have students turn to S52 in their books. The first problem is a SOLVE problem. You are only going to complete the S step with students at this point. Tell students that during the lesson they will learn how to solve problems involving constant speed and unit pricing with tape diagrams. They will use this knowledge to complete this SOLVE problem at the end of the lesson **{SOLVE, Verbal Description}**

Unit Rate Problems - Concrete with Tape Diagrams**(CP, WG, M, GP) S52 (Answers on T116.)**

M, GP, WG, CP: Have students turn to S52 in their books. Distribute 5 index cards, 10 counters, and 5 toothpicks to each student pair. Make sure students know their designation as Partner A or Partner B. **{Verbal Description, Concrete Representation}**

*Teacher Note: In this lesson, the ratios will be written with a fraction bar – example $\frac{2}{1}$.

MODELING**Unit Rate Problems - Concrete with Tape Diagrams**

Step 1: Direct students' attention to S52 and use the following instructions to complete this step.

- Partner A, place 1 index card on the workspace.
- Partner B, create a set with 2 counters and 1 toothpick.
- Identify the **ratio** of the counters to the toothpicks. $(\frac{2}{1})$

Step 2:

- Partner A, place another index card beside the first one to create a strip of index cards. Tape the index cards to join them together. This is called a **tape diagram**.

- Partner B, create another set of counters and toothpicks exactly like the set in Step 1.
- Partner A, identify the ratio on this index card. $(\frac{2}{1})$
- Partner B, identify the ratio of both sets together. $(\frac{4}{2})$

Step 3:

- Continue to have students build the sets until they have made 5, identifying the ratio for each set and for the sets altogether.

Step 4:

- Have student pairs examine the tape diagram.
- Partner A, was each set exactly the same? (Yes)
- Partner B, did the sets grow at a constant rate? (Yes) Justify your answer. (We kept adding identical sets to the tape diagram.)
- Partner A, at what rate were the toothpicks added to the counters? $(\frac{2}{1})$
- This is called the **unit rate**.

*Teacher Note: Have students leave the tape diagram on the workspace.

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Unit Rate Problems – Pictorial with Tape Diagrams

(CP, WG, M, GP, IP) S53, S54, S55 (Answers on T117, T118, T119.)

M, GP, CP, WG:

Have students turn to S53 in their books. Make sure students know their designation as Partner A or Partner B. {Verbal Description, Concrete Representation, Pictorial Representation, Graph, Graphic Organizer}

MODELING**Unit Rate Problems - Pictorial with Tape Diagrams**

Step 1: Direct students' attention to Problem 1 and use the following instructions to complete this step.

- Partner A, explain how to draw a picture of the first index card of your tape diagram in the space provided.
- Have all students draw the first index card.
- Partner B, explain how to draw a picture of the second index card next to the first index card in the space provided.
- Have all students draw the second index card.
- Complete the tape diagram drawing with your partner.

Step 2: Have students look at the table below the tape diagram.

- Partner A, what is the label for Row One in the table? (counters)
- Partner B, what is the label for Row Two of the table? (toothpicks)
- Partner A, what is the number of counters in the first set? (2) Record on the table.
- Partner B, what is the number of toothpicks in the first set? (1) Record on the table.

Step 3: Have student pairs complete the table using the pictorial representation of the tape diagram.

- Partner A, are the ratios from left to right equivalent? (Yes.)
- Partner B, at what rate were the toothpicks added to the counters? ($\frac{2}{1}$)

Step 4: Have students look at the coordinate plane below the table.

- Partner A, if the counters represent the x-axis, what is the x-coordinate of the first ratio? (2)
- Partner B, if the toothpicks represent the y-axis, what is the y-coordinate of the first ratio? (1)
- With your partner, plot the first set of coordinates on the coordinate plane.
- Have student pairs graph the remaining points from the table and then connect the points using a ray.

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- Partner A, how many counters are there for 5 toothpicks? (10) Record.
- Partner B, does the tape diagram show a constant ratio of counters to toothpicks? (Yes) Record.
- Partner A, does the coordinate plane show a constant ratio of counters to toothpicks? (Yes) Record
- Partner B, at what rate were the toothpicks added to the counters? ($\frac{2}{1}$) Record.

IP, CP, WG:

Have students work with their partners to complete Problems 2 and 3 on S54 and S55. Point out to students that the tape diagram on S55 uses the “index cards,” but instead of a picture, words are on the cards to indicate the unit rate on each card. Have students come back together as a class and share their results. {**Verbal Description, Graph, Pictorial Representation, Graphic Organizer**}

Unit Rate Problems with Unit Pricing and Constant Speed

(CP, WG, M, GP, IP) S56, S57 (Answers on T120, T121.)

M, GP, CP, WG:

Have students turn to S6 in their books. Make sure students know their designation as Partner A or Partner B. {**Verbal Description, Concrete Representation, Pictorial Representation, Graph, Graphic Organizer**}

MODELING**Unit Rate Problems with Unit Pricing and Constant Speed**

Step 1: Direct students’ attention to Problem 1 on S56.

- Partner A, identify what the tape diagram shows. (a ratio of 70 pages to 3 hours) Record on the table in the first box.
- Partner B, what is the number of pages and hours for the second set of data on the table? (140 pages to 6 hours.) Record.

Step 2: Have student pairs complete the table.

- Partner A, does the tape diagram show a **constant rate** of pages read? (Yes) Record.
- Partner B, at what rate were the pages read? ($\frac{70}{3}$) Record.
- Partner A, what do we know from the rate that is given? (70 pages read every 3 hours)
- Partner B, identify what you know. (number of pages read every 3 hours) Record.
- Partner A, what is the value we need to find or the unknown? (number of pages read in 18 hours) Record.

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Step 3: Look at the graphic organizer.

- Partner A, what is the rate for the data? ($\frac{70}{3}$) Record in the first box.
- Partner B, what does the 70 represent? (pages read)
- Partner A, what does the 3 represent? (hours it takes to read 70 pages)
- Partner B, what is unknown? (pages read in 18 hours) Record.

Step 4: Have student pairs discuss how they can use the rate they know to determine how many pages are read in 18 hours. (Answers may vary, but can include making equivalent fractions.)

- Partner A, for the equivalent fractions, identify what fractional part of the ratio is known. (hours it takes to read a number of pages) Record that as the denominator.
- Partner B, what is the unknown fractional part of the equivalent fraction? (the pages read)
- Partner A, what can we use to represent the unknown? (A **variable** – use x for the numerator.) Record.

Step 5: Have student pairs discuss what operation can be used to determine the missing value in the numerator.

- Partner B, what operation did we perform on the 3 to find 18 in the denominator to make the values equivalent? (Multiply by 6.)
- Partner A, explain the next step to create an equivalent fraction. (Multiply the numerator by 6.)
- Partner B, explain why we can multiply the first fraction by the fraction of $\frac{6}{6}$. ($\frac{6}{6}$ is equal to 1, and when we multiply any factor by 1 the product is the original factor.)
- Partner A, determine the numerator for the equivalent fraction if 70 is multiplied by 6. (420) Record.
- Record the solution. (420 pages in 18 hours)

Step 6: Direct students' attention to Problem 2.

- Partner B, explain how Problem 2 is different from Problem 1. (It does not have the picture or the table, but it does have a graphic organizer.)
- Partner A, identify the **unit pricing**, and explain to your partner what it means. ($\frac{20}{6}$ means 20 cents for 6 pencils). Record the rate.
- Partner B, identify the unknown. (pencils purchased for 80¢) Record.

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- Record the rate as the first fraction in the equivalent fraction box.
- Partner A, what does the 20 represent? (amount of money for 6 pencils)
- Partner B, what does the 6 represent? (number of pencils)

Step 7: Partner A, what is the unknown fractional part of the equivalent fraction? (the number of pencils that can be purchased for 80¢) Record as an x for the denominator.

- With your partner, determine what operation you would perform on the 20 to find 80 in the numerator to make the values equivalent. (Multiply by 4.)
- Partner B, determine the denominator for the equivalent fraction if 6 is multiplied by 4. (24) Record.
- Record the solution. (80 cents for 24 pencils)

Step 8: Direct students' attention to Problem 3.

- Partner B, explain how Problem 3 is different from Problem 2. (It does not have a graphic organizer.)
- Partner A, identify the rate, and explain to your partner what it means. ($\frac{2}{3}$, which means \$2.00 for 3 notebooks.)
- Partner B, identify the unknown and set up an equivalent fraction. (Cost of 12 notebooks, $\frac{2}{3} = \frac{x}{12}$)
- Have student pairs solve for the answer, using the same factor to multiply for denominator and numerator. ($\frac{2}{3} = \frac{8}{12}$, which means \$8.00 for 12 notebooks.) Record.

IP, CP, WG:

Have students work with their partners to complete Problems 4 – 6 on S57. Monitor closely to make sure students are using the appropriate vocabulary. Have students come back together as a class and share their results. {Verbal Description, Graphic Organizer}

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SOLVE Problem**(GP, WG) S58 (Answers on T122.)**

Remind students that the SOLVE problem is the same one from the beginning of the lesson. Complete the SOLVE problem with your students. Ask them for possible connections from the SOLVE problem to the lesson. (Students will work with rate problems involving unit pricing and constant speed using tape diagrams.)

{SOLVE, Verbal Description, Graphic Organizer}

If time permits...**(IP, I, CP) S59 (Answers on T123.)**

Have students complete Questions 1 - 7 on page S59.

[CLOSURE]

To wrap up the lesson, go back to the essential questions and discuss them with students.

- How can I define rate? (*A rate is a special ratio that compares two quantities that can be measured using different units, such as miles per hour, or the cost of a quantity of an object.*)
- How can I use a tape diagram to show ratios? (*The tape diagram repeats the unit ratio, which can be collected as sets of data.*)
- How can I show a constant rate using a tape diagram? (*The tape diagram shows the repetition of the unit rate, which is the same, steady, constant set represented over and over. It does not change.*)

[HOMEWORK] Assign S60 for homework. (Answers on T124.)

[QUIZ ANSWERS] T125 – T127

The quiz can be used at any time as extra homework or to assess how students progress on solving rate problems involving unit pricing and constant speed including using tape diagrams.