LESSON 33: Bivariate Data and Relative Frequency

[OBJECTIVE]
The student will use bivariate categorical data to construct and interpret two-way tables that will summarize the data collected, and use frequencies and relative frequencies to describe possible relationships between the two variables.

[PREREQUISITE SKILLS]
interpreting data tables, surveys, scatter plots

[MATERIALS]
Student pages **S430 – S444**
Calculators

[ESSENTIAL QUESTIONS]
1. How can a two-way table help to interpret patterns of association in bivariate categorical data?
2. What is meant by the term relative frequency in relationship to bivariate data in a two-way table?
3. How can the relative frequency be used to explain the association of two categories in a two-way table?

[WORDS FOR WORD WALL]
frequency, relative frequency, two-way table, univariate, bivariate, survey

[GROUPING]
Cooperative Pairs (CP), Whole Group (WG), Individual (I)
*For Cooperative Pairs (CP) activities, assign the roles of Partner A and Partner B to students. This allows each student to be responsible for designated tasks within the lesson.

[LEVELS OF TEACHER SUPPORT]
Modeling (M), Guided Practice (GP), Independent Practice (IP)

[MULTIPLE REPRESENTATIONS]
SOLVE, Verbal Description, Algebraic Formula, Graphic Organizer, Table

[WARM-UP] (IP, I, WG) S430 (Answers on T891.)
Have students turn to S430 in their books to begin the Warm-Up. Students will complete percent problems to prepare for finding relative frequency with bivariate data. Monitor students to see if any of them need help during the Warm-Up. Have students complete the problems and then review the answers as a class. **{Algebraic Formula}**

[HOMEWORK]
Take time to go over the homework from the previous night.

[LESSON] [3 days (1 day = 80 minutes) - (M, GP, IP, WG, CP)]

SOLVE Problem (WG, GP)	**S431 (Answers on T892.)**

Have students turn to S431 in their books. The first problem is a SOLVE problem. Students are going to complete all steps of the SOLVE problem using prior knowledge of reading the table and finding percents. Tell students that during the lesson they will learn how to interpret bivariate data and create two-way tables. Review SOLVE problem as a class. **{SOLVE, Verbal Description, Graphic Organizer, Table}**

Discovery Activity – Exploring Univariate and Bivariate Data
(M, GP, WG, CP) S432, S433 (Answers on T893, T894.)

WG, M, CP, GP:	Have students turn to page S432 in their books. Assign the roles of Partner A and Partner B. **{Verbal Description, Graphic Organizer, Table}**

MODELING

Discovery Activity – Exploring Univariate and Bivariate Data

Step 1: Direct students' attention to Question 2 at the top of S432. Ask partners to discuss Questions 2 and 3 and then be prepared to share answers as a whole class.
- Partner A, how many variables were involved in the chart? (1) Record.
- Partner B, explain your answer. (The problem asked you to identify the percentage of students who could retake the test.) Record.
- Partner A, what other data situations did you discuss with your partner that use only 1 variable? (finding mean, median, MAD, IQR, data questions about bar graphs, line graphs, histograms, box plots, dot plots, etc.)

Step 2: Direct students' attention to Questions 4 and 5.
- Partner A, how many students' grades were in the table. (24)
- Partner B, is it possible to identify from the table any reason as to why the students had the grade they did? (No)

Step 3: Have partners compare the table in Question 4 and the table in Question 6 and discuss how they are different.
- Partner B, what is one way that the two tables are different? (four columns instead of two) Record.
- Partner A, identify another difference in the tables. (Study time was added to the second table.) Record.
- Partner B, describe how the display of the scores is different in the second table. (The scores were divided into two groups.) Record.
- Partner A, how many variables are displayed in the second data table. (2) Record. This is called a **two-way table.**
  Partner B, identify the two variables (grade and time studied) Record.

	Studied an average of 3 hours	Studied an average of 1 hour	Total
Grade of 71% and above	14	4	18
Grade of 70% or below	1	5	6
Total	15	9	24

Step 4: Direct students' attention to the top of page S433. When we compare the tables from Question 4 and Question 6 from S432, we can make some observations, draw some conclusions, and chart them in the graphic organizer.
- Partner A, what was the purpose of the chart in Question 4? (describe or identify) Record.
- Partner B, what was the purpose of the chart in Question 6?(explain the relationship between the two variables) Record.
- Partner A, does the chart in Question 4 show any possible cause for the grades? (No) Record.
- Partner B, does the chart in Question 6 show any possible cause for the grades? (Yes) Record.
- Partner A, what type of graph could be used to display the data in the table for Question 4? (bar graph, dot plot, histogram) Record.
- Partner B, what type of graph could be used to display the data in the table for Question 6? (two-way table) Record.

	Single variable	Two variables
Purpose	**describe or identify**	**explain relationship**
Shows relationship?	**no**	**yes**
Types of graphs	**bar graph, dot plot, histogram**	**two-way table**
Type of Data	**univariate**	**bivariate**

Step 5: Direct students' attention to 7a below the table.
- Have student pairs discuss the picture by 7a.
- Partner A, can you identify the type of cycle pictured below 7a? (unicycle)
- Partner B, based on that picture, what can you conclude about the meaning of the prefix "uni"? (It means one.)
- Partner A, explain the meaning of the word variable. (letter that represents a value)
- Partner B, when we combine the prefix "uni" with the base of "variate," what do you think the word **univariate** means? (Data with a single variable) Record univariate in 7a.

Step 6: Direct students' attention to 7b below the table.
- Have student pairs discuss the picture by 7b.
- Partner A, can you identify the type of cycle pictured below 7b? (bicycle)
- Partner B, based on that picture, what can you conclude about the meaning of the prefix "bi" (It means two.)
- Partner A, explain the meaning of the word variable. (letter that represents a value)
- Partner B, when we combine the prefix "bi" with the base of "variate", what do you think the word **bivariate** means? Record bivariate in 7b.

Step 7: Direct students' attention to Question 8.
- Partner A, based on the discussion from Question 7, what is the definition of univariate data? (data that describes one variable) Record.
- Partner B, based on the discussion from Question 7, what is the definition of bivariate data? (data that describes two variables) Record.

Bivariate Data and Relative Frequency with SOLVE
(M, GP, CP, IP, WG) S434 (Answers on T895.)

M, GP, WG, CP :   Have students turn to S434 in their books. Students will work with bivariate data and relative frequency using SOLVE. Make sure students know their designation as Partner A or Partner B. **{Verbal Description, SOLVE, Graphic Organizer, Table}**

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Bivariate Data and Relative Frequency with SOLVE

Step 1: Have students turn to S434 in their books.

- Students will work in student pairs to read the SOLVE problem and complete the S and O steps.

- Give students a few minutes to complete S and O and then go over the answers as a whole group.

- Partner A, what is the problem asking me to find (the percent of students who scored 71% or above, that studied for 3 hours.) Record.

- Partner B, identify the necessary facts. (information given in the two-way table) Record.

Step 2: Have student pairs discuss the information found in the table and identify which values they will need to use.
- Partner A, what would be the first step for our plan? (Identify the value that shows the students who had a grade of 71% and above and studied an average of 3 hours.)
- Partner B, what would the second step of our plan be? (Divide the number of students who had a grade of 71% and studied 3 hours by the total number of students who received a grade of 71% or above.) Record.
- Partner A, what would our operation or operations be? (division)
- Partner B, explain how we know that division will be the operation we will use. (We know that we need to find the percent of a number, and we will need to divide the two values in order to find the percent.) Record the operation.

	Studied an average of 3 hours	Studied an average of 1 hour	Total
Grade of 71% and above	14	4	18
Grade of 70% or below	1	5	6
Total	15	9	24

Step 3: Partner A, determine an estimate for Step V. (about 80%) Record.
Partner B, complete the plan. 14 ÷ 18 = 0.78 = 78% (Record.)

IP, CP, WG: Have students work in pairs to complete the E step for the SOLVE problem. Then come back together as a class and share their results. **{Algebraic Formula, Verbal Description, Graphic Organizer}**

Bivariate Data and Relative Frequency – Extending SOLVE
                    (M, GP, CP, IP, WG) S434, S435 (Answers on T895, T896.)

M, GP, WG, CP :   Have students turn to S435 in their books. Students will work with bivariate data and relative frequency extending the SOLVE problem from S434. Make sure students know their designation as Partner A or Partner B. **{Verbal Description, SOLVE, Graphic Organizer, Table}**

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Bivariate Data and Relative Frequency – Extending SOLVE

Step 1: Have students turn to S435 in their books.
- Partner A, identify the one question we answered about the data in the table on S434. (What percent of the students who scored 71% or above, studied an average of 3 hours?)

Step 2: Direct students to use the table below Question 6.
- Partner A, what was the first value we had to identify? (students who scored 71% or above and studied an average of 3 hours.) Record.
- Partner B, how many students had studied an average of 3 hours and received a grade of 71% or higher? (14) Record. Explain how you determined this answer. (I identified the row with the first variable of the grade. This has two values – one for the students who studied 3 hours and one for the students who studied one hour. I identified the value of the students who had studied 3 hours.)

Step 3: This value tells us how many times something happened during the time period.
- Partner A, when we want to talk about how often something happens, is there another word we can use? (how frequent it is)
- Partner B, if we refer back to Question 2 and how many students had studied an average of 3 hours and received a grade of 71% or higher, and we want to identify how often that happened in our data set, what could we ask for? (the frequency of the event)
- Partner A, what term can be used to identify this value? (**frequency**) Record.
- Partner B, what was the second value we had to identify? (the total number of students who scored a grade of 71% or above) Record.
- Partner A, how many total students scored a grade of 71% or above? (18) Record.

Step 4: Partner B, what word do we use to describe all the people in our family or the people we are related to? (relatives)
- Partner A, can you give an example and explain what that means? (There is some relationship between them. Examples will vary: mother-daughter, etc.)
- We can also identify relationships with the numbers in our table. If we combine the term that describes how often an event happens (frequency) in relationship (relatives) to the total possible trials, we have the relative frequency.
- When the students compare those who scored a grade of 71% or above and studied 3 hours to the total number who scored 71% or above, we create a ratio which displays the **(relative frequency)**. Record.

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	Studied an average of 3 hours	Studied an average of 1 hour	Total
Grade of 71% and above	14	4	18
Grade of 70% or below	1	5	6
Total	15	9	24

Step 5: We can also extend our analysis of the two-way table to include describing possible associations or correlations between the two variables.
- We know that 14 out of 18 or (78%) of the students who scored 71% or above, studied 3 hours. Record.
- Partner B, what percentage of students who scored a 70% or below had spent 3 hours studying? (1 ÷ 6 = 16.7%) Record.
- Have student pairs discuss Questions 9 and 10 and then share answers as a whole group. Answers may vary but suggested answers are provided for Questions 9 and 10. It is important that whatever conclusion the students make, they are able to explain their thinking and support their conclusion.

Bivariate Data and Relative Frequency – Using Two-Way Tables to Interpret Data   **(M, GP, CP, IP, WG) S436, S437, S438 (Answers on T897, T898, T899.)**

M, GP, WG, CP: Have students turn to S436 in their books. Students will work to complete two-way tables and interpret bivariate data. Make sure students know their designation as Partner A or Partner B. **{Verbal Description, Graphic Organizer, Table}**

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Bivariate Data and Relative Frequency – Using Two-Way Tables to Interpret Data

Step 1: Have students turn to S436 in their books. Look at the table to identify the variables.
- Partner A, identify the variable in the first row. (how much time was spent shopping – more than 2 hours or less than 2 hours)
- Partner B, identify the second variable which is in the first column (how much money was spent – more than $50 or less than $50)

	Shopped for more than 2 hours	Shopped for less than 2 hours	Total
Spent more than $50	7	1	**8**
Spent less than $50	2	5	**7**
Total	**9**	**6**	**15**

Step 2: Partner A, what are the two values in the row identified as those who spent more than $50? (7 and 1)
- Partner B, identify the label on the last column. (Total) What does the word total mean? (that values must be combined)
- Partner A, when we combine 7 and 1, what operation are we using? (addition)
- Partner B, what is the sum of 7 + 1? (8) Record in the Total column.

Step 3: Partner A, what are the two values in the row identified as those who spent less than $50? (2 and 5)
- Partner B, identify the label on the last column. (Total) What does the word total mean? (that values must be combined)
- Partner A, when we combine 2 and 5, what operation are we using? (addition)
- Partner B, what is the sum of 2 + 5? (7) Record in the Total column.
- Partner A, if we add the two values in the Total column, what is the sum? (8 + 7 = 15) Record the sum.

Step 4: Partner B, what are the two values in the column identified as those who shopped for more than 2 hours? (7 and 2)
- Partner A, identify the label on the bottom row. (Total) What does the word total mean? (that values must be combined)
- Partner B, when we combine 7 and 2, what operation are we using? (addition)
- Partner A, what is the sum of 7 + 2? (9) Record in the Total row.

Step 5: Have student pairs work together to complete the process from Step 4 to determine the total for the column labeled, "Shopped for less than two hours."

Step 6: Partner A, what do you notice about the sum of the Total row? (It is the same as the sum of the Total column.) Can you explain why that happens? (The total number of friends at the mall is 15. When we look at each characteristic - time spent, money spent - both must equal the total number of friends. For example: the friends either spent more than $50 or less than $50 so the number 15 would include everyone.)
- Partner B, what can you conclude from that information? (When the vertical values are added and the horizontal values are added in the Total column and Total row, the sum must be equal.)

Step 7: Partner A, what percent of the people in the **survey** shopped for more than 2 hours? (60%) Record. Explain how you determined that answer. (I divided the total number of people who shopped for more than 2 hours by the total number of people who were surveyed. 9 ÷ 15 = 0.6 = 60%)

- Partner B, what percent of the people in the survey spent more than $50? (53%) Record. Explain how you determined that answer. (Divided the total number of people who spent more than $50 by the total number of people who were surveyed. 8 ÷ 15 = 0.53 = 53%)
- Is there a relationship between the amount of time spent shopping and the amount of money spent? (Yes.) Record. Explain your thinking. (More than half of the people shopped for more than 2 hours and spent more than $50.)
- Are there other possible relationships that can be identified from the data in the table? (Answers will vary, but students must be able to support their answer using the data.)

IP, CP, WG: Have students work in pairs to complete the two-way tables and answer the questions on bivariate data on S436, S437, and S438. Remind students that all conclusions about the data must be supported from the data in the table. Then come back together as a class and share their results. **{Verbal Description, Graphic Organizer, Table}**

Bivariate Data and Relative Frequency – Creating Two-Way Tables to Interpret Data   (M, GP, CP, IP, WG) S439, S440, S441 (Answers on T900, T901, T902.)

M, GP, WG, CP: Have students turn to S439 in their books. Students will work to create two-way tables and interpret bivariate data. Make sure students know their designation as Partner A or Partner B. **{Verbal Description, Graphic Organizer, Table}**

MODELING

Bivariate Data and Relative Frequency – Creating Two-Way Tables to Interpret Data

Step 1: Have students turn to S439 in their books.
- We have learned how to interpret the data and make conclusions based on information in a two-way table. Now let's learn how to create a two-way table when given data in a list or a word problem.
- Have student pairs read the word problem and discuss the steps to create the table.
  A survey of 8th graders at a local middle school found that 54 students owned some type of video game system, and 40 of those also have a computer. There are 12 students who do not have a video game system but do have a computer. Four students do not own either a computer or a video game system.

Step 2: Partner A, what is the first step to create the table? (deciding how many columns and rows) Record.
- Partner B, how do you determine the number of columns and rows? (Read the problem and identify the number of characteristics you will need to show in the table.) Record.
- Partner A, how many characteristics are in this problem? (Two: students who own or do not own video game systems, students who have or do not have a computer)

Step 3: Talk to your partner and discuss what will be the next step to create the table. (Determine which characteristic will be the x-variable and which will be the y-variable.)
- Let's use the video game system as the x-variable.
- Partner B, where will we place it and why? (Place the video game system or no video game system in the first row because the x-axis is the horizontal axis whenever we are graphing.)
- Partner A, what is the second variable? (students who have a computer or do not have a computer)
- Partner B, where will we place it and why? (Place those labels in the first column because the y-axis variable is on the vertical axis to stay consistent with the coordinate plane.)

Step 4: Direct students' attention to S440.
- Partner A, how many students own a video game system? (54 total students own a video game system.) Record.
- Partner B, how many of those students have a computer? (40 of those 54 have a computer.) Record.
- Partner A, how many students have a video game system but no computer? (54 − 40 = 14) Record.

- Partner B, how many students have no video game system, but do have a computer? (12 students have no video game system, but do have a computer.) Record.
- Partner A, how many total students have a computer? (40 + 12 = 52) Record.
- Partner B, Is there another fact given? (4 students do not own either a video game or a computer.) Record.
- Partner A, how many total people were surveyed? (70) Record.

	Video game system	No video game system	Total
Computer	40	12	52
No computer	14	4	18
Total	54	16	70

Step 4: How do I know if my values are correct? The vertical total must be equal to the horizontal total. (54 + 16 = 70; 52 + 18 = 70) Record.

Step 5: Record all information in the table on S440.

IP, CP, WG: Have students work in pairs to complete the two-way tables and answer the questions on bivariate data on S441. Then come back together as a class and share their results. **{Verbal Description, Graphic Organizer, Table}**

SOLVE Problem   **(GP, WG) S442 (Answers on T903.)**

Have students turn to S442 in their books. The SOLVE problem is on bivariate data. Complete the SOLVE problem with your students. Ask them for possible connections from the SOLVE problem to the lesson. Students will use their knowledge of bivariate data and relative frequency to solve this problem. **{SOLVE, Graphic Organizer, Verbal Description, Table}**

If time permits...   **(IP, CP) S443 (Answers on T904.)**

Have student pairs work together to complete the survey project.

[Closure]

To wrap up the lesson, go back to the essential questions and discuss them with students.

- How can a two-way table help to interpret patterns of association in bivariate categorical data? *(The relationship between the two variables can be identified by the values in the table.)*

- What is meant by the term relative frequency in relationship to bivariate data in a two-way table? *(The term relative frequency describes the relationship between the partial value and the total value in a category in the two-way table.)*

- How can the relative frequency be used to explain the association of two categories in a two-way table? *(The relative frequency can be written as a percent based on the ratio between the two values.)*

[Homework] Assign S444 for homework. (Answers on T905.)

[Quiz Answers] **T906–T907**

The quiz can be used at any time as extra homework or to see how students progress on interpreting bivariate data and relative frequency and creating data displays with bivariate data.