LEGGON 4: Rational Numbers in the Real Number System

[OBJECTIVE]
The student will determine the square roots of perfect squares, and categorize rational numbers in the real number system and apply this understanding to solve mathematical and real world problems.

[PREREQUISITE SKILLS]
fractions, decimals, percents

[MATERIALS]
Student pages S29–S40
Student Algebra Tiles (16 yellow tiles per student pair)
Overhead Algebra Tiles
Sticky Notes
Colored paper
Painter’s tape
Algebra tiles

[ESSENTIAL QUESTIONS]
1. Explain how to determine if a real number is rational.
2. How do you evaluate the square root of a perfect square? Justify your answer using a model.
3. Use variables to explain how a ratio can be used to write a rational number. Defend your thinking.

[WORDS FOR WORD WALL]
rational numbers, square roots, perfect squares, natural numbers, whole numbers, integers, terminating decimals, repeating decimals, counting numbers, radical symbols

[GROUPING]
Cooperative Pairs (CP), Whole Group (WG), Individual (I)
*For Cooperative Pairs (CP) activities, assign the roles of Partner A or Partner B to students. This allows each student to be responsible for designated tasks within the lesson.

[LEVELS OF TEACHER SUPPORT]
Modeling (M), Guided Practice (GP), Independent Practice (IP)

[MULTIPLE REPRESENTATIONS]
SOLVE, Verbal Description, Pictorial Representation, Concrete Representation, Graphic Organizer

[WARM-UP] (IP, I, WG) S29 (Answers on T78.)
• Have students turn to S29 in their books to begin the Warm-Up. Students will apply knowledge of fractions, decimals and percents to translate from one form to another. Monitor students to see if any of them need help during the Warm-Up. After students have completed the warm-up, review the solutions as a group. {Graphic Organizer}

[HOMWORK]
Take time to go over the homework from the previous night.

**LESSON** [1-2 days (1 day = 80 minutes) - M, GP, WG, CP, IP]

### SOLVE Problem

Have students turn to S30 in their books. The first problem is a SOLVE problem. You are only going to complete the S step with students at this point. Tell students that during the lesson they will learn how to determine the square roots of perfect squares, determine the decimal equivalent of rational values and categorize rational numbers. They will use this knowledge to complete this SOLVE problem at the end of the lesson. *(SOLVE, Verbal Description, Graphic Organizer)*

### Square Roots – Concrete and Pictorial

(M, GP, WG, CP) S30, S31 (Answers on T79, T80.)

**GP, M, CP, WG:** Pass out the yellow algebra unit tiles. Make sure students know their designation as Partner A or Partner B. Use the following activity to model the concept of perfect squares and square roots using algebra tiles. *(Concrete Representation, Pictorial Representation, Algebraic Formula, Verbal Description)*

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**MODELING**

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**Square Roots – Concrete and Pictorial**

**Step 1:** Have students work together in pairs to create a square from four yellow algebra unit tiles. Have students model and explain their square and then have each partner draw the square in the space on their own page.

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**Step 2:** Partner A, what is the length of each side of the square? (2 units) Record.

**Step 3:** Ask students to use 9 tiles to make a square and then draw the square in the space.

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**Step 4:** Partner B, what is the length of each side of the square? (3 units) Record.
**Step 5:** Have students turn to S31 in their books.
- Partner A, how do you find the area of a square? (Multiply side by side, or \( s^2 \).) Record.

**Step 6:** Partner B, what is the area of the first square on S30? (4 square units) Record.
- Partner A, what is the area of the second square on S30? (9 square units) Record.
- Partner B, what do you need to do to one side of the square to get the area? (square it) Record.
- Partner A, explain what it means to square a number. (Multiply the number times itself.) Record.

**Step 7:** Partner B, what is the opposite of addition? (subtraction) Record.
- Partner A, when we talk about the opposite operation, explain why the operations are opposite. (We can use subtraction to undo an addition problem or to determine if we have the correct sum.) Record.
- We can also use the opposite of squaring a number.
- Partner B, what is the opposite of squaring a number? (finding the square root) Record.
- Ask students if they know what the symbol for a square root is. Show students the radical symbol that is used when they want to find the square root of a number. Record.

**Step 8:** Partner A, if we look at our examples from S30 we can see that \( 2^2 = 4 \), what do you think the value of \( \sqrt{4} \) is? (2) Record.
- Partner B, if \( 3^2 = 9 \), then what is the value of \( \sqrt{9} \)? (3) Record.

**Step 9:** Have students make a square using 16 yellow algebra unit tiles, then draw the square in the space.

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**Step 10:** Ask students to find the square root of 16. (4) Record.
Have student pairs discuss and then explain their own definition of a square root. Students can share their answers with the whole group. (The square root of a number is a number that when multiplied by itself will yield the number under the radical.) Record.
LESSON 4: Rational Numbers in the Real Number System

MODELING
The Hierarchy of Rational Numbers

Step 1: Have students look at the blank graphic organizer on page S32.
- Partner A, what information do you see in the organizer? (different number values; There is also a word bank below the organizer.)
- Explain to students that they will be exploring different types of numbers and using the graphic organizer on S32 as well as the wall chart to categorize the numbers.
- Have student pairs look at the list of numbers on the top of S33 and discuss what the values in the list have in common. (Answers may vary, but possible answers include: all values are less than or equal to 7 and greater than or equal to -6. All the values can be plotted on a number line.)

Step 2: Direct students’ attention to the colored tiles drawn for Question 2 on S33.
- Partner A, how many tiles are there total? (5 yellow unit tiles) Record.
- Partner B, explain how you determined the number of tiles. (I counted them.) Record.
- Partner A, what number did you start with when you began counting in your head? (1) Record. Explain your thinking. (When we count items, we begin with the number 1, then 2, then 3, then 4 and continue on until we have counted all the items.) Record.

Step 11: The square roots that we have found on S31 and S32 are called (perfect squares) because each number’s square root is a whole number. Record.

The Hierarchy of Rational Numbers

M, GP, WG, CP: Use the following activity to introduce students to categorizing rational numbers. The class will be completing a wall sized graphic organizer to help build a model for the hierarchy of rational numbers. Students will be completing a similar organizer on S32. To prepare for this activity, you can use painter’s tape or colored sheets of paper to create the blank template from S32 on a wall or whiteboard for students to complete. Pass out a sticky note to each student. {Concrete Representation, Verbal Description, Pictorial Representation, Graphic Organizer}
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Step 3: Have student pairs look at the list of words in the word bank on the bottom of S32 to identify which of the words may be used to describe numbers that we count with.

- Partner B, since we use these numbers to count, what would be a good term to use to identify this group of numbers? (Counting Numbers) Record. They are also known by another name: (Natural Numbers) Record.
- Partner A, describe where these numbers are located in the graphic organizer on S32. (in the smallest box) Record.
- Partner B, list the values you see in that box. (1, 2, 3, 4, 5, 6, 7, 8, 9, 10...) Record.
- Partner A, explain what is meant by the three dots that follow the value of 10. (It means that we can continue on with the list to infinity with the Counting or Natural numbers.) Record.
- Have students add the label of Counting or Natural Numbers to the smallest box on S32.
- Partner B, what are some of the characteristics you notice about the Counting Numbers? (Answers may vary, but may include: positive values, begin at 1, do not include 0, counting by ones to infinity.) Record.
- Partner A, which of the values in Question 10 would be classified as a counting number? (7) Record.

Step 4: Have students count off by fours. Have each student who is a Number 1 write a counting number on their sticky note and place it in the category titled Natural or Counting Numbers on the wall.

Step 5: Direct students’ attention to the top of S34 and describe the tiles drawn in the box for Question 1. (There are 5 yellow or positive tiles.)

- Partner B, if we took away 5 yellow tiles from above, what number would we use to represent how many tiles we have? (0) Record.
- Partner A, is zero a counting number? (No) Record.
- Partner B, explain your thinking? (Counting numbers begin with 1, and do not include 0.) Record.

Step 6: Have student pairs look at the list of words in the word bank on the bottom of S32 to identify which of the words may be used to describe numbers that include 0 and all the positive numbers.
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• Partner A, what do you think would be a good term to use when identifying these numbers? (Whole Numbers) Record. Explain your thinking. (All the values are positive but also include 0.)
• Partner B, describe where these numbers are located in the graphic organizer on S32. (They are located in the box outside of the Counting Numbers box because all the numbers in the Counting Number box are also Whole Numbers.) Record.
• Partner A, list the values you see in that box. (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10…) Record.
• Partner B, explain what is meant by the three dots that follow the value of 10. (It means that we can continue on with the list to infinity with the Whole numbers.) Record.
• Have students add the label of Whole Numbers to the box outside of the Counting Number box on S32.
• Partner B, what are some of the characteristics you notice about the Whole Numbers? (Answers may vary, but may include: positive values, begin at 0, counting by ones to infinity.) Record.
• Partner A, which of the following values would be classified as whole numbers? (0, 7) Record.

Step 7: Have each student who is a Number 2 write a Whole Number on their sticky note and place it in the category titled Whole Numbers on the wall.

Step 8: Have students turn to page S35.
• Direct students’ attention to the colored tiles drawn below Question 1.

R R R R R

• Partner B, explain how these tiles are different from the counters on S32. (They have the letter R which represents a negative value instead of a positive value.) Record.
• Partner A, what is the value of the tiles? (-5) Record.
• Partner B, does the value represented above with tiles fall into one of the two categories we have discussed already? (No) Record.
• Partner A, explain your thinking. (Negative values are not included in Natural or Whole Numbers.) Record.

Step 9: Have student pairs look at the list of words in the word bank on the bottom of S32 to identify which of the words may be used to describe numbers that include both positive and negative values.
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- Partner B, when we learned about different operations using the red and yellow counters, what name did we use to describe positive and negative whole numbers? *(Integers)* Record.
- Partner A, explain where these numbers are located in the graphic organizer on S32. (They are located outside of the Whole Numbers box because all the values in the Whole Numbers box are also Integers.) Record.
- Partner B, list the values you see in that box. (...-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...) Record.
- Partner A, explain what is meant by the three dots that start and end the list of values. (It means that we can continue on with the list to infinity in either direction, toward the positive or negative.) Record.
- Have students add the label of Integers to the box outside of the Counting Number Box on S32.
- Partner B, what are some of the characteristics you notice about the Integers? (Integers are all the Whole Numbers and their negative partners.) Record.
- Partner A, which of the following values would be classified as integers? (...-6, 0, 7) Record.

**Step 10:** Have each student who is a Number 3 write an Integer on their sticky note and place it in the category titled Integers on the wall.

**Step 11:** Direct students to the top of S36.
- Partner A, identify the last three values from our list on S33. (1.25, 0.\(\frac{3}{5}\), \(\frac{4}{5}\)) Record.
- Partner B, do we have a category for these numbers? (No) Record. Explain your thinking. (Our categories only involve numbers that are counted in whole increments and not parts like fractions and decimals.) Record.
- Partner A, what term can we use to describe the relationship between the 4 and 5 in the fraction \(\frac{4}{5}\)? (Writing the fraction is one of three ways to write a ratio. We can also write this relationship as 4:5 or 4 to 5.) Record.

**Step 12:** Have student pairs look at the list of words in the word bank on the bottom of S32 to identify which of the words may be used to describe numbers that include values that are not integers.
- Partner B, based on the graphic organizer on page S32, what category of numbers can we place the fraction in to explain that it falls under the category of ratios? *(Rational Numbers)* Record.
• Partner A, if our newest category includes ratios, how can we explain rational numbers? (A rational number is any number that can be written as a fraction. We can use variables to represent the numerator and denominator \( \frac{a}{b} \) to show that the values can be any integers. \( b \) cannot be equal to zero.)

• Partner B, our graphic organizer shows that Integers, Whole Numbers and Natural or Counting Numbers are all under the category of Rational Numbers. Is this correct? Explain your thinking and defend your answer. (Yes, because all of these categories of numbers can be represented as a ratio or fraction in the form of \( \frac{a}{b} \). For example, \( 5 = \frac{5}{1} \), \( 0 = \frac{0}{1} \) and \( -2 = \frac{-2}{1} \).) Record.

• Partner A, list the values you see in the box for Rational Numbers. \((1.25, 0.\overline{3}, \frac{4}{5})\) Record.

• Partner B, which two values are not written as a ratio? \((1.25, 0.\overline{3})\) Record.

• Partner A, explain why these two decimals are rational numbers. [They can be written as a ratio using integers: \( 1.25 = \frac{125}{100}, 0.\overline{3} = \frac{1}{3} \)] Record.

• Have students add the label of Rational Numbers to the box outside of the Integer Box.

**Step 13:** Have each student who is a Number 4 write a Rational Number on their sticky note and place it in the category titled Rational Numbers on the wall.

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**Rational Numbers – Decimal Expansion (M, GP, CP, WG) S37 (Answers on T86.)**

**M, GP, CP, WG:** Have students turn to S37 in their books. Use the following activity to help students explore decimal expansion for rational numbers. Make sure students know their designation as Partner A or Partner B. {Verbal Description, Graphic Organizer}

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**MODELING**

**Rational Numbers – Decimal Expansion**

**Step 1:** Direct students’ attention to the top of S37.

- Have students share what they know about converting fractions and decimals from our warm-up. (We can easily change back and forth between the two using division to create decimals and using place value to create fractions.)
- Partner A, what is the first number in the graphic organizer? \( \left( \frac{1}{2} \right) \)
- Partner B, explain how we find the decimal form of this fraction. (Divide the numerator by the denominator.)
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- Partner A, is the decimal form of this fraction repeating or terminating? (terminating) Explain your thinking. (The fraction value is exact because the division problem terminates or ends with a quotient that has a remainder of zero.) Record.
- Partner B, identify the next number? \( \left( \frac{4}{6} \right) \)
- Partner A, explain how we find the decimal form of this fraction. (Divide the numerator by the denominator.)

*Teacher Note: The division of the fractions that are in the graphic organizer can be modeled in a variety of ways based upon your students’ needs. Some students may only need to see the first fraction divided out to see that it repeats and then they can work in student pairs to determine the decimal for the other two fractions in the chart. If there are students who need more support, you can model all three division problems that are in the chart.

Step 2:  
- Partner B, at what decimal do you arrive when you divide? (0.66666...) Record.
- Partner A, is this decimal terminating or repeating? (repeating) Record. Explain your thinking. (The decimal value is approximate because the division problem does not have a quotient that comes out with a remainder of zero. The number 6 continuously repeats beyond the decimal.) Record.

Step 3: Have students complete the two remaining rows in the graphic organizer. Review the answers as a whole group.

Step 4: Direct students’ attention to Question 1 below the graphic organizer.
- Partner A, what do you notice about the decimals of the rational numbers? (They either stop, or they continue on with a repeating pattern.) Record.
- Partner A, what type of decimal is the equivalent of \( \frac{2}{5} \)? (terminating decimal) Record.
- Partner B, explain this. (There is a point where the quotient comes out evenly with no remainder.) Record.
- Partner A, what type of decimal is the equivalent of \( \frac{4}{9} \)? (repeating decimal) Record.
- Partner B, explain your thinking. (When you divide 4 by 9, it repeats the same number over and over in the quotient. We write the repeating portion of the decimal with a bar over it. Therefore, for \( \frac{4}{9} \) it should be written as 0.\( \bar{4} \) because the 4 will repeat continuously.) Record.
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**SOLVE Problem** *(WG, CP, IP) S38 (Answers on T87.)*

Remind students that the SOLVE problem is the same one from the beginning of the lesson. Complete the SOLVE problem with your students. Ask them for possible connections from the SOLVE problem to the lesson. (Students have worked with identifying rational values and finding the decimal expansion for rational values.)

{SOLVE, Verbal Description, Graphic Organizer}

**If time permits...** *(CP, IP) S39 (Answers on T88.)*

Have students complete the blank organizer template to categorize numbers that fall in the Real Number System.

[CLOSURE]

To wrap up the lesson, go back to the essential questions and discuss them with students.

- Explain how to determine if a real number is rational. *(All rational numbers can be represented as a decimal that terminates or repeats.)*
- How do you evaluate the square root of a perfect square? Justify your answer using a model. *(Create a square using the number of algebra tiles defined in the perfect square. The side of the square you create will be the square root because that value times itself will equal the original number.)*
- Use variables to explain how a ratio can be used to write a rational number. Defend your thinking. *(All rational numbers can be written in the form of a ratio using the variables a and b, \( \frac{a}{b} \), where a and b are both integers and \( b \neq 0 \).)*

[HOMEWORK] Assign S40 for homework. (Answers on T89.)

[QUIZ ANSWERS] T90–T91

The quiz can be used at any time as extra homework or to see students progress with understanding the categories of rational numbers and their decimal expansions.